Approximate Bayesian Computation (ABC) for Bilinear Processes *

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The class of bilinear models ([4]) plays an important role in modeling non-linearity for various reasons, such as the fact that it is an obvious generalization of ARMA models. Under fairly general conditions, bilinear processes approximate finite order Volterra series expansions to any desired order of accuracy over finite time intervals. Volterra series expansions are a dense class within the class of nonlinear time series. Therefore, bilinear processes are also a dense class within nonlinear processes, approximating any nonlinear process to a desired level of accuracy. Due to their capacity of producing clusters of large values, bilinear models are often suggested. However, they are not frequently used in practice due to inference problems.

In general, the conditions of stationarity and invertibility cannot be written in terms of the model parameters in a easily verifiable form. The likelihood cannot be written explicitly, and particularly for heavy-tailed data, conditional least squares and quasi-likelihood methods do not seem to give good results. However, it is relatively easy to simulate such series.

Approximate Bayesian Computation (ABC) algorithms arise as ways to deal with inference problems associated with likelihood functions which are analytically difficult to handle or even intractable (see for instance [5]). As such, establishing an ABC protocol for such nonlinear systems seems to be a good idea. The major challenge is to find a set of statistics capable of representing the nonlinear dynamics of the system. In this work, we suggest seven statistics, namely a portmanteau statistic that captures the linear time dynamics through the empirical autocorrelations, the Moments’ estimator ([3]), which quantifies the tail heaviness and the extremal index (see e.g. [2]) that measures the degree of clustering of large values. The implemented method is based on the recently proposed method by [1] which used $k$-nearest neighbor techniques. We apply these strategies to a simple first order bilinear model for different independent and identically distributed innovation processes.

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References


